6. G. I. Danilova, O. B. Ioffe, V. A. Dyundin, et al., "Heat transfer during boiling of liquids at heater surfaces with capillary-porous coatings," in: Temperature Field and Hydraulics of Steam Generators [in Russian], Nauka, Leningrad (1978), pp. 73-78.

VAPOR CONDENSATION ON PLATE DURING HORIZONTAL MOTION OF COOLANT

V. F. Stepanchuk* and V. V. Kozhushko

UDC 536.24

An analytical study is made of heat transfer during vapor condensation on a vertical plate. An engineering method of verifying the heat transfer calculations is also shown.

We consider the problem of heat transfer during condensation of saturated vapor on a vertical plate while a coolant moves transversely relative to the descending condensate film. The same assumptions will be made here as those on which the derivation of the widely known Nusselt equation [1] is based. We will also assume that no stirring of the coolant occurs on the cooled side and that its temperature varies only in the direction of its flow. The latter condition is possible only in the case of laminar flow with a small vertical temperature gradient or with the stream subdivided into a large number of parallel channels.

The problem will be formulated as follows: it is required to determine the magnitude of heat transfer, the thickness of the condensate film, and the temperature of the coolant when on one side of the vertical plate there is vapor under pressure p_s (saturation temperature t_s) and on the other side the plate is wetted by a horizontal stream of coolant moving through a channel of width z at velocity w. The physical characteristics of the coolant are known and the temperature t_0 of the coolant at the entrance to this heat exchanger is given. This formulation of the problem is illustrated in Fig. 1.

The elementary quantity of heat expended on heating the coolant is

$$dQ = w\rho c_p z \frac{\partial t}{\partial y} dxdyd\tau = W \frac{\partial t}{dy} dxdyd\tau. \tag{1}$$

The same quantity of heat can be determined from the equation of heat transfer

$$dQ = K(t_s - t)\, dxdyd\tau = \frac{\lambda}{a+\delta}(t_s - t)\, dxdyd\tau, \tag{2}$$

where

$$K = \frac{1}{\dfrac{1}{\alpha} + \left(\dfrac{\delta}{\lambda}\right)_w + \left(\dfrac{\delta}{\lambda}\right)_f} = \frac{\lambda}{a+\delta}; \tag{3}$$

$$a = \left[\frac{1}{\alpha} + \left(\frac{\delta}{\lambda}\right)_w\right]\lambda. \tag{4}$$

The parameter a characterizes both the thermal impedance and the heat transfer from wall to coolant.

Finally, according to the method used for deriving the Nusselt equation [1], the change in the rate of condensate flow in the film is

$$dm = \frac{g(\rho' - \rho'')}{3\nu} \frac{\partial \delta^3}{\partial x}. \tag{5}$$

The quantity of transferred heat will then be

$$dQ = r \frac{g(\rho' - \rho'')}{3\nu} \frac{\partial \delta^3}{\partial x}. \tag{6}$$

*Deceased.

Belorussian Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 4, pp. 539-543, April, 1982. Original article submitted February 5, 1981.

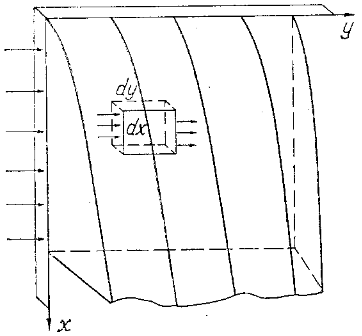

Fig. 1

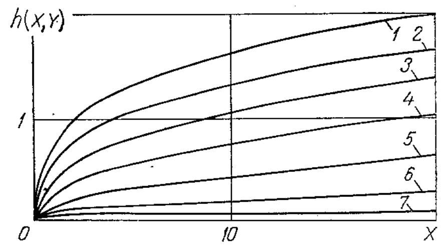

Fig. 2

Fig. 1. Derivation of the system of differential equations.

Fig. 2. Thickness of condensate film as function of the dimensionless coordinates X and Y: 1) Y = 0; 2) 1; 3) 1.5; 4) 2; 5) 2.5; 6) 3; 7) 3.5.

Equating expressions (1), (2), and (6), we obtain a system of two differential equations describing the heat transfer process

$$W\frac{\partial t}{\partial y}=\frac{\lambda}{a+\delta}(t_s-t)=\frac{rg(\rho'-\rho'')}{3\nu}\frac{\partial\delta^3}{\partial x}. \tag{7}$$

The boundary conditions are stipulated on the basis of the following considerations. At x = 0 the thickness of the condensate film is zero, namely $\delta(0, y) = 0$, while $t(0, y)$ satisfies the differential equation

$$W\frac{dt(0,\ y)}{dy}=\frac{\lambda}{a}(t_s-t(0,\ y)),$$

whose solution is

$$t(0,\ y)=t_s-(t_s-t_0)\exp\left(-\frac{\lambda}{Wa}\,y\right). \tag{8}$$

At y = 0 the temperature of the coolant remains constant $t(x, 0) = t_0$ and $\delta(x, 0)$ is found from the equation

$$\frac{\lambda}{a+\delta(x,\ 0)}(t_s-t)=\frac{rg(\rho'-\rho'')}{3\nu}\frac{d\delta^3(x,\ 0)}{dx},$$

whose integral can be expressed as

$$a\delta^3(x,\ 0)+\frac{3}{4}\delta^4(x,\ 0)=\frac{3\nu\lambda(t_s-t_0)}{rg(\rho'-\rho'')}x. \tag{9}$$

Upon changing in expressions (7), (8), and (9) to dimensionless variables θ, h, X, and Y, we arrive at the boundary-value problem

$$-\frac{\partial\theta}{\partial Y}=\frac{\theta}{1+h}=\frac{\partial h^3}{\partial X}, \tag{10}$$

$$\text{at } X=0\ h(0,\ Y)=0,\ \theta(0,\ Y)=\exp(-Y), \tag{11}$$

$$\text{at } Y=0\ h^3(X,\ 0)+\frac{3}{4}h^4(X,\ 0)=X,\ \theta(X,\ 0)=1, \tag{12}$$

for the dimensionless temperature $\theta(X, Y)$ and film thickness $h(X, Y)$. For solving the system of Eqs. (10) we eliminate from it $\theta(X, Y)$ so that, after a few simple transformations, there remains the equation

$$\frac{\partial}{\partial X}\left[\frac{\partial}{\partial Y}\left(h^3+\frac{3}{4}h^4\right)+h^3\right]=0$$

for $h(X, Y)$. After integration for the boundary conditions (11) and (12), we have for $h(X, Y)$ the transcendental equation

$$h\exp h=h_0\exp h_0\exp(-Y/3), \tag{13}$$

where $h_0 = h(X, 0)$ satisfies condition (12):

$$h_0^3+\frac{3}{4}h_0^4=X. \tag{14}$$

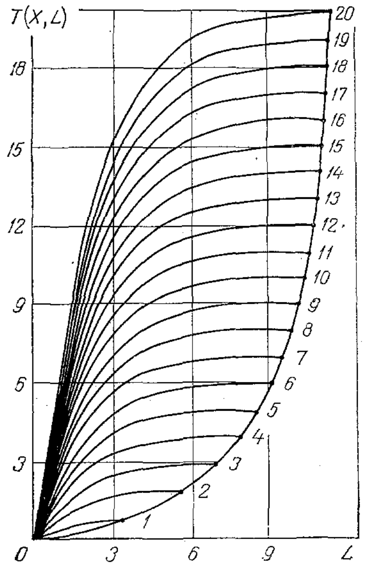

Fig. 3. Dependence of the dimensionless integral T(X, Y) on the condenser length and height; numbers at the points indicate values of X, while Y = 13X/(X + 2.5).

The system of Eqs. (13) and (14) was solved by a numerical method. The algebraic equation (14) was analyzed by the combination method of chords and tangents, whereupon h(X, Y) was calculated by the iteration method with h_0 known. In order to ensure beforehand a convergent iteration process, it is expedient to transform Eq. (13) into [2]

$$h = 0.9 + 0.1h_0 \exp h_0 \exp(-h) \exp(-Y/3). \tag{15}$$

For the purpose of establishing the intervals of the dimensionless variables X and Y for these calculations, we estimated these quantities. According to these estimates, the maximum values of these dimensionless quantities are related to x and y through the equalities

$$Y = 10y, \; X = 10x. \tag{16}$$

Inasmuch as a real condenser is x = 0.1-3 m high and y = 0.1-0.6 m long, most expedient for a numerical analysis will be the intervals of the dimensionless variables

$$X = 1-30, \; Y = 1-60. \tag{17}$$

The results of calculations, made with the aid of a computer, reveal that already at Y > 6 the thickness of the condensate film becomes h(X, Y) = 0 (temperature of the coolant close to the temperature of the condensate). This important fact must be taken into account in condenser design and performance calculations. The results of calculations are shown graphically in Fig. 2.

Now, knowing the thickness of the condensate film at each point (x, y), we can calculate the quantity of heat transferred during condensation

$$Q = r\frac{g(\rho' - \rho'')}{3\nu}\int_0^l \delta^3 dy, \tag{18}$$

where l is the length of the condenser, or in dimensionless variables

$$Q = \frac{W(t_s - t_0)}{k_x}\int_0^L h^3 dY, \tag{19}$$

where

$$k_x = \frac{3\nu\lambda(t_s - t_0)}{a^4 rg(\rho' - \rho'')}; \; k_y = \frac{\lambda}{Wa}; \; L = k_y l. \tag{20}$$

Relation (19) indicates that, with the geometrical and operating parameters of the condenser given, calculation of the quantity of heat reduces to evaluation of the dimensionless integral

$$T = \int_0^L h^3 dY. \tag{21}$$

This integral was evaluated on a computer according to the Simpson rule. The results of these calculations (graphically shown in Fig. 3) indicate that, when L is sufficiently large, T(X, L) and, thus, also the quantity of heat released during condensation becomes almost equal to X without further changing as L continues to increase. In the first approximation one can propose the following relation between X and L_{max}

$$L_{max} = \frac{13X}{X + 2.5}, \tag{22}$$

where L_{max} is the value of L from which on the difference between T(X, L) and X will be smaller than 0.1 without reservations. This means that in designing a laminate condenser with a prescribed height X one must select its length from the condition

$$L < L_{max} = \frac{13X}{X + 2.5}, \tag{23}$$

since exceeding this critical length L_{max} will not result in a higher condensate flow rate.

In the case of known geometrical and operating parameters one calculates the quantity of transferred heat (verifying calculation) in the following sequence: 1) parameter a is evaluated according to expression (4); 2) one calculates the dimensionless coordinate $X = k_x x$ for the given condenser height x, with k_x found from expression (20); 3) one calculates the dimensionless coordinate $Y = k_y y$ for the given condenser length y (when this length y is not given, then the dimensionless length L is selected from condition (23)), 4) one evaluates the dimensionless integral T(X, L) according to relation (21) with the aid of the graph in Fig. 3 for given X and L; 5) one calculates the quantity of heat Q transferred during condensation according to relation (19).

NOTATION

x, y, coordinates in the directions of condenser height and length respectively; z, channel width; w, velocity of the coolant; ρ, density of the coolant; c_p, specific heat at constant pressure; t_0, initial temperature; t, temperature of the coolant; δ, thickness of the condensate film; t_s, saturation temperature; λ, thermal conductivity of the condensate film; ν, kinematic viscosity of the condensate; r, heat of phase transition; $W = w\rho c_p z$, water equivalent; $\theta = (t_s - t)/(t_s - t_0)$, dimensionless temperature of the coolant; $h = \delta/a$, dimensionless thickness of the condensate film; $Y = y\lambda/Wa$, dimensionless length; and $X = 3\nu\lambda(t_s - t_0)/a^4 rg(\rho' - \rho'')$, dimensionless width.

LITERATURE CITED

1. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, Heat Transmission [in Russian], Energiya, Moscow (1975).
2. V. M. Levshits and B. F. Litvin, Approximate Calculations and Programming on "Nairi-2" Computer [in Russian], Mashinostroenie, Leningrad (1977).